

Hydromagnetic Flow Dynamics with Power-Law Velocity Over a Stretching Surface

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ABSTRACT

In this analysis, a power law velocity distribution boundary layer flow over a stretching sheet in the presence of the transverse magnetic field is studied. Non-dimensional equations are transformed into a nonlinear ordinary differential equation by using similarity transformations and also a special form magnetic field is chosen the numerical solutions are obtained by MATLAB in built solver bvp4c. The effects of the nondimensional parameter on velocity profile are discussed through graphs and skin friction coefficient results are presented in tabular form and found the good agreement with previously results.

Keywords: Magnetic Field, Power Law velocity, bvp4c, Skin friction, boundary layer.

I. INTRODUCTION

The two-dimensional boundary layer flow caused by a linear stretching sheet in an otherwise quiescent fluid, exponential solution a very simple closed form, and the solution of the associated linear heat conduction equation was first discussed by Crane [1]. This kind of issue is of enthusiasm for the assembling of sheeting material through an expulsion procedure. Both the basic flow problems and the heat transfer issue have since been reached out in different ways. For example, Afzal and Varshney [3] and Banks [4] have considered the more general case of the sheet stretching with a power-law velocity, i.e. $U(x) = ax^m$ where a and m are constants. The solutions have been

studied for the range $-2 < \beta < \infty$ where the parameter $\beta = \frac{2m}{1+m}$. It was found that when $\beta = -2$ no solution

was possible but between $-2 < \beta < 2$, solution exist with $a > 0$ whereas, for $2 < \beta < \infty$, a must be negative. In addition, apart from the case $m = -1$, all solutions exhibited exponential decay. Chakrabarti and Gupta [5] extended the linear stretching problem to studied the effect of a constant transfer magnetic field. Makinde and Aziz [6] studied the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Boundary-layer flow of a nanofluid past a stretching sheet was studied by Khan and Pop [7]. Rasekhet al [8] given a numerical solutions for a nanofluid past over a stretching circular cylinder with non-uniform heat source. Syahira Mansur et al [9] analysed the three-dimensional flow and heat transfer past a permeable exponentially stretching/shrinking sheet in a nanofluid. Natural convective boundary-layer flow of a nanofluid past a vertical plate was analyzed by Kuznetsov and Nield [10]. Radiative heat transfer in a hydromagnetic nanofluid past a non-linear stretching surface with convective boundary condition was described by Rahman and Eltayeb [11]. Acharya et al [12] studied the heat and mass transfer over an accelerating surface with heat source in presence of suction and blowing. Effects of the thermal radiation on the boundary layer flow over an exponentially stretching sheet in the presence of viscous dissipation were discussed by BSGoud et al [13]. A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition was analysed by Abdul Aziz [14].

In this paper, the analysis made about the boundary layer flow of a Newtonian flow which is carried about by a stretching sheet as indicated by a power law velocity distribution in the presence of magnetic field.

II. GOVERNING EQUATIONS

As far as possible the governing equations for the steady two-dimensional flow of an electrically conducting fluid in the existence of a magnetic field $B(x)$ are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots(1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(x)}{\rho} u \quad \dots(2)$$

where u, v are the velocity flow components in the x - and y -directions respectively; ν is the kinematic viscosity, ρ the fluid density, and σ is the electrical conductivity. It is assumed that the induced magnetic field is negligible, the external electric field is zero and the electric field due to polarization of charges is also negligible.

The appropriate boundary conditions are given by

$$\begin{aligned} u &= U_w(x); \quad v = 0, \quad \text{at } y \rightarrow 0 \\ u &\rightarrow 0, \quad \text{as } y \rightarrow \infty \end{aligned} \quad \dots (3)$$

As in Afzal [1] and Banks [2], introduce the following change of variables,

$$\psi = \sqrt{\frac{2\nu x U(x)}{1+m}} f(\eta), \quad \eta = y \sqrt{\frac{(1+m)U(x)}{2\nu x}} \quad \dots (4)$$

By defining the stream functions as $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ equation (1) can be easily verified that the continuity

equation (1) is identically satisfied. Similarity solutions exist if we assume that $U(x) = ax^m$ and the magnetic field B has the special form $B(x) = B_0 x^{(m-1)/2}$... (5)

Using stream functions, (3), and (4) in eqns. (2) transform to the following form:

$$f''' + \beta f f'' - f'(\beta f' + M) = 0 \quad \dots (6)$$

The boundary conditions (3) becomes in the following form:

$$\begin{aligned} f &= 0, \quad f' = 1, \quad \text{at } \eta \rightarrow 0 \\ f' &\rightarrow 0, \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad \dots (7)$$

Where $\beta = \frac{2m}{1+m}$, $M = \frac{2\sigma B_0^2}{(1+m)\rho a}$ (m, a are constants).

III. SOLUTIONS OF THE PROBLEM

The nonlinear ordinary differential equation (6) along with boundary conditions (7) are incorporated with the help of MATLAB tool bvp5c. To get this, the set of ordinary differential equations are first transformed to first order ordinary differential equations by using the successive substitutions

$$\begin{aligned} f &= f_1, \quad f' = f_2, \quad f'' = f_3 \\ f_1' &= f_2, \quad f_2' = f_3, \quad f_3' = f_2(\beta f_2 + M) - (f_1 * f_3) \end{aligned}$$

The boundary conditions take the following structure

$$f_1(0) = 0, \quad f_2(0) = 1, \quad \eta \rightarrow 0; \quad f_2(\eta_\infty) = 0, \quad \eta \rightarrow \infty$$

The asymptotic boundary condition (7) at the margin was stable to 10^{-6} . In this approach, the choice of $\eta_\infty = 3$, in agreement with standard practice in the boundary layer analysis.

IV. RESULTS AND DISCUSSION

Table 1 records a few estimations of $f''(0)$ for $\beta = 1.5$ and 5.0 when the magnetic parameter takes different values as appeared. The third column gives estimations of $f''(0)$ determined from (6) using the shooting method and the last column gives esteems registered by a direct numerical combination of condition (7) subject to conditions (8) utilizing a MATLAB in built in solver bvp4c strategy. In Table 2, we demonstrate the comparing results for $\beta = -1.0$ and -1.5 .

It very well may be seen from Table 1 that excellent outcomes were obtained by utilizing in built solver for almost every one of the cases. For larger estimations of the Magnetic parameter M , the estimations of $f''(0)$ are for all intents and purposes indistinguishable to those acquired utilizing a bvp4c solver. The varieties of f' with η over the limit layer for $\beta = 1.5$ and $M = 0.0, 1.0, 5.0, 10.0$ are appeared in Fig. 1. When all is said in done, the impacts of the magnetic field are to build the skin friction and in the meantime decreases the thickness of the boundary layer (see also Fig. 3). For negative estimations of β as shown in Table 2, in built solver bvp4c is again astounding if M is more prominent than about 0.5 for $\beta = -1.0$ and about 1.0 for $\beta = -1.5$. Be that as it may, for low estimations of M , it gives wrong outcomes. The explanation behind this can most likely be seen from Fig. 2 where plots of f' versus η are appeared. For estimations of M smaller than about 0.5 , the conduct of f' close to the surface is not quite the same as the standard cases, as in f' increases to a most extreme before it begin to diminish. In Fig. 3, the plots demonstrate the variety of $f''(0)$ with the magnetic parameter M . For $\beta = -1.5$ and for a smaller range of M almost zero, there is a decrease of skin friction as M increases. Notwithstanding, in the various cases, it is seen that expanding the attractive field quality builds the skin friction. This increase is sharper for negative estimations of β , particularly when M is in the approximate range of 0.5 and 5 .

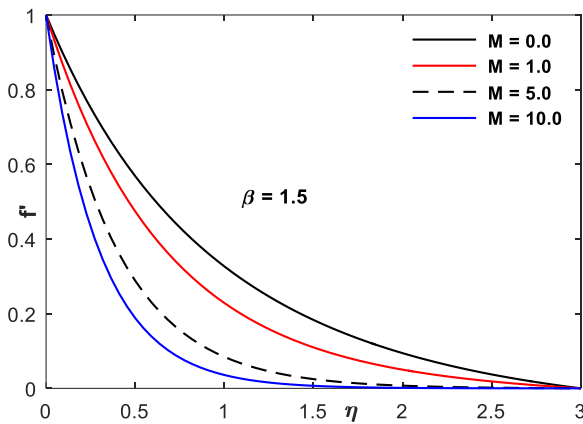


Fig. 1. Plots of f' vs η for $\beta = 1.5$.

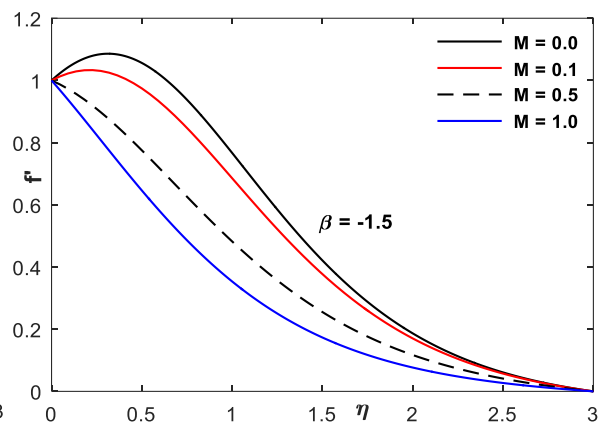


Fig. 2. Plots of F' vs η for $\beta = -1.5$.

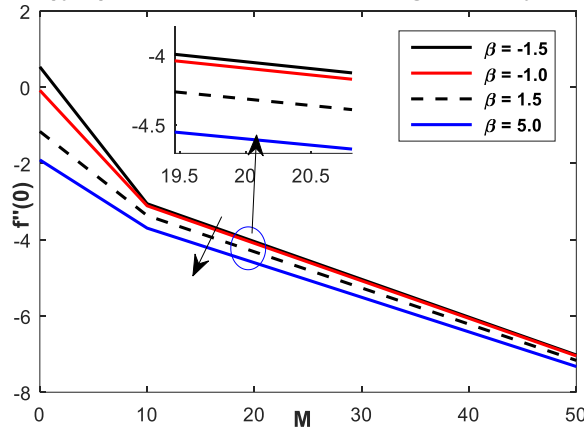


Fig. 3. Plots of $f''(0)$ vs M for $\beta = -1.5, -1.0, 1.5$ and 5.0

Table 1. Comparison of values of $f''(0)$ for $\eta = 1.5, 5.0$ and various values of M

η	M	shooting method	Present study
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1.5	0	-1.1486	-1.148649
	1	-1.52527	-1.525276
	5	-2.51615	-2.51616
	10	-3.366331	-3.366328
	50	-7.16471	-7.164734
	100	-10.0664	-10.066444
5	0	-1.90253	-1.90255
	1	-2.15289	-2.152894
	5	-2.94144	-2.941513
	10	-3.69566	-3.695702
	50	-7.32561	-7.325823
	100	-10.1816	-10.181668

Table 2. Comparison of values of $f(0)$ for $\beta = -1.0, -1.5$ and various values of M

η	M	Shooting method	Present study
-1.0	0	0	-0.004869
	0.1	-0.13215	-0.135447
	0.2	-0.24783	-0.249996
	0.3	-0.35006	-0.351445
	0.4	-0.4414	-0.442281
	0.5	-0.52395	-0.52451
	1	-0.85111	-0.851176
	5	-2.16287	-2.162867
	10	-3.11003	-3.110023
	100	-9.98335	-9.983305
-1.5	0	0.72725	0.53458
	0.1	0.45107	0.312184
	0.2	0.23038	0.132752
	0.3	0.05203	-0.016168
	0.4	-0.09506	-0.142954
	0.5	-0.21922	-0.253261
	1	-0.65298	-0.660562
	5	-2.08524	-2.085243
	10	-3.05623	-3.056226
	100	-9.96665	-9.966611

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