

## Development of a Mathematical Model to Optimize Electricity Production through Biogas-Firewood Substitution in Boilers

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### ABSTRACT

Biogas produced by the treatment of industrial and urban waste become an important alternative to produce renewable energy. It can be used in generating thermal and electric energy and can reduce environmental impacts in production processes, increasing energy efficiency. In the present paper it was developed a model to predict the amount of savings achieved when substituting partially firewood by biogas in industrial boilers to produce electricity. Annual costs and consumption of firewood, and electricity tariffs are considered in the model. A case study in a biotechnology industry located in Maringá, Brazil, was used to test the model, in which biogas is produced from waste water organic sludge and is composed of 73.93 % of methane. Results show that, for the case studied, savings of 26 % were obtained, when compared to the consumption of firewood currently used. The potential of electricity generation is 5 % in relation to the total consumption of the industry. This model could be used for different types of wastewater treatment for biogas production and other fuel used in industrial boilers.

**KEYWORDS:** biogas; electricity; renewable energy; mathematical modeling.

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### 1. INTRODUCTION

Biogas can be produced using anaerobic digestion technologies, with different substrates, pretreatments, reactor configurations and operational concerns [1]. In the anaerobic digestion it can be used the locally available residual biomass from various sources (animal waste, domestic sewage, industrial wastewater, agricultural waste, etc) [2]. Because of this, biogas can be considered carbon neutral once the carbon in biogas comes from organic matter (feedstocks), that captured this carbon from the atmospheric CO<sub>2</sub> over relative short timescale [3]. In addition to the environmental results, biogas production provides significant reductions in unwanted odor from wastewater treatments stations, transport and final destination of sludge from physical-chemical treatments.

Through specific processes, it is possible to enrich the methane content in the biogas, once it is an indication of higher energy content of the fuel [4]. According to Hosseini and Wahid [5], high concentrations of CO<sub>2</sub> in biogas components increases CO formation and weakens the flame. It means that [6] higher CO<sub>2</sub> concentrations decrease the potential heat intensity.

Although biogas heating value (about 3000 – 6000 kcal/m<sup>3</sup>) is lower when compared to natural gas or liquefied petroleum gas it can be used [7] for different purposes, as the production of heat and electricity (cogeneration), heat only, electricity only, fuel for vehicles and as raw material in the chemical industry [8, 9]. It can be used for the self-consumption in industrial plants and, in some cases, the excess of the produced electricity can be sold, exported and compensated by connection to the power distribution network.

Among all the advantages of producing electricity from biogas, it is possible to emphasize that the non-centralized generation will be close to the loading points, from a renewable source that was previously treated as waste, allowing an extra result and reducing the amount of electricity purchased from the energy company. The use of biogas for electricity generation, besides bringing benefits to the environment, also helps in waste management.

This work has as main objective the development of a mathematical model to evaluate the capacity of electricity production using biogas when burned in boilers in partial replacement of firewood or any other fuel for the production of steam.

A case study was used to apply the proposed model. The energetic equivalence of the biogas was studied, when used as a source of thermal and electric energy. The capacity of biogas production, the potential of electricity generation, the costs of implementation and the return of the investment were calculated.

## 2. MATERIALS AND METHODS

### Model Development

A mathematical model was developed to evaluate the capacity of electricity generation from burning biogas obtained from the treatment of residual biomass in anaerobic reactors in the partial substitution of other biomass. The biogas production is evaluated from the estimated influent Chemical Oxygen Demand (COD) load to the reactor, which is converted into methane gas [10, 11]. With the theoretical production of methane determined, the total biogas production can be estimated from the expected content of CH<sub>4</sub>.

Knowing the heating value, it is possible to determine the available potential for the production of electric power (*Pot*) and estimate the annual production of electricity in the plant (*E*) [12].

In the model, it is considered the energy conversion efficiency of 33 % in stationary engines of Otto cycle and internal combustion [13].

The model allows the calculation of the average biogas flowrate available for the generation of electricity and the best route for the use of biogas, i.e., the production of thermal energy or the production of electricity, can be evaluated.

The amount of biogas to be produced can be estimated from the Chemical Organic Demand (COD) in the anaerobic reactor, converted into methane:

$$COD_{CH_4} = Q(S_0 - S) - Y_{obs}QS_0 \quad (1)$$

In this equation,  $COD_{CH_4}$  is COD load converted in methane gas (kgCOD<sub>CH<sub>4</sub></sub>/d),  $Q$  is the average influent flowrate (m<sup>3</sup>/d),  $S_0$  is the influent COD concentration (kgCOD/m<sup>3</sup>),  $S$  is the effluent COD concentration (kgCOD/m<sup>3</sup>) in the anaerobic reactor and  $Y_{obs}$  is the coefficient of solids production in the system, in terms of COD (0.11 to 0.23 kgCOD<sub>sludge</sub>/kgCOD<sub>appl</sub>).

The methane mass (kgCOD<sub>CH<sub>4</sub></sub>/d) volumetric production (m<sup>3</sup>CH<sub>4</sub>/d) can be calculated by (2):

$$Q_{CH_4} = COD_{CH_4}/K(t) \quad (2)$$

where:  $Q_{CH_4}$  is the volumetric methane production (m<sup>3</sup>/d) and  $K(t)$  is given by:

$$K(t) = PK_{COD}/RT \quad (3)$$

Where  $P$  is the atmospheric pressure,  $K_{COD}$  is the COD corresponding to one mole of CH<sub>4</sub>,  $R$  is the gas constant and  $T$  is the reactor operational temperature.

The volumetric flow rate of produced biogas (m<sup>3</sup>/d) can be calculated by (4):

$$Q_{biogas} = Q_{CH_4}/C_{CH_4} \quad (4)$$

where:  $C_{CH_4}$  is the methane concentration (%) in the biogas.

The available potential for the generation of electric power of the plant (*Pot*) is obtained by equation (5):

$$Pot = Q_{biogas} \square LHV \quad (5)$$

In this equation,  $\square$  is the energy conversion efficiency [12] and  $LHV$  is the calorific power expressed in kW [13]. If the amount of methane contained in biogas is 60 %, this heating value is equivalent to 5,500 kcal/m<sup>3</sup>

[14]. According to Salomon and Lora [15], for other percentage values, it could vary from 5,374 to 5,971 kcal/m<sup>3</sup>.

The electric power produced annually (*E*), expressed in kWh, can be calculated by:

$$E = Pot \Delta t \quad (6)$$

In this equation,  $\Delta t$  is the annual number of hours of plant operation (h/y).

In accordance with Andriani et al. [16] with 1 m<sup>3</sup> of biogas it is possible to achieve 1.25 kWh.

The heating value of firewood developed by Brand et al. [17] is a function of the moisture and storage span presented in Table I.

TABLE I. Variation of the heating value (medium for *Eucalyptus dunnii* logs according to the harvesting season and storage season) and the moisture content.

Storage span (months)	Moisture content (%)	Heating value (kcal/kg)
0-1	54	1,646
2	43	2,204
4	34	2,624
6	32	2,746

The capital cost ( $I$ ) necessary to implement the potential biogas energy for each source and for biogas plants powered by internal combustion engines is given by equation (7):

$$I = aP + b \tag{7}$$

Where  $P$  is the power (kW) and  $a$  and  $b$  are constants proposed by Santos et al. [18].

To evaluate the amount of electricity produced from the biogas, the following steps are proposed:

*Step One*

The heating value of the fuel used as a source to produce thermal energy which will be replaced by biogas need to be known. Also, it must be known the concentration of methane contained in the biogas to determine its heating value. From this relation, the percentage of fuel replaced with the use of biogas can be obtained.

*Step Two*

With known values of fuel cost per unit mass, electricity consumption and the tariff systems values it is possible to estimate the amount biogas used. The systems for the generation of electricity using biogas must operate in conditions that provide the greatest possible cost reduction, whether by fuel substitution or energy costs.

*Step Three*

From the amount of biogas generated by the wastewater treatment system it is determined the energy equivalence based on the electricity produced from 1 m<sup>3</sup> of biogas. With the plant average energy consumption and with the potential of electricity to be produced, the percentage of energy produced with biogas and the avoided cost can be determined.

*Step Four*

It must be considered the biogas filtering system, the internal combustion engine and the executive connection design in the investment costs for the generation of electricity. To evaluate the return of the investment, the system maintenance costs are deducted according to the lifetime of the equipment.

*Step Five*

Finally, the following results are obtained:

The fuel consumed for the thermal energy generation (kg/y); the fuel consumed jointly with biogas in the boiler for the thermal energy generation (kg/y); the amount of fuel replaced by biogas for the thermal energy generation (%); the annual cost avoided with the use of biogas for the replacement of part of the fuel in the thermal energy

generation; the electric power consumption (kWh/y); the potential production of electricity with the use of biogas (kWh/y) and the replacement of electricity with the use of biogas (%).

The return of the investment ( $RI$ ) can be calculated:

$$RI = I/ACAEBP \tag{8}$$

Where  $ACAEBP$  is the avoided annual cost of Electricity with Biogas used.

### 3. CASE STUDY

The mathematical model developed to estimate the amount of electricity produced by replacing part of the fuel with biogas was applied in a case study in an agroindustry located in Maringá, Brazil. The company aims using biogas as a source of thermal energy in partial replacement of firewood in the burner to produce steam for the industrial process.

The input data for the model, collected in the company were the Composition of the biogas in terms of CH<sub>4</sub> content to evaluate the heating value; the wastewater flow daily; the organic load in the anaerobic reactor - COD<sub>influent</sub> and COD<sub>effluent</sub>; the consumption of firewood in the boiler with use of biogas; the consumption of

firewood in the boiler without the use of biogas; the cost of firewood; the annual consumption of electricity and the electricity tariff cost.

**4. RESULTS AND DISCUSSIONS**

**Biogas Composition**

Table II presents the average composition of the biogas used in this study measured in the year of 2015 and the first and second semesters of year of 2016.

*TABLE II. Biogas Composition.*

Component	2015	1/2016	2/2016	Average
CH <sub>4</sub> (%)	75.4	65.4	81.0	73,93
CO <sub>2</sub> (%)	20.5	18,5	18.1	19.03
O <sub>2</sub> (%)	0.0	0,2	0.1	0.06
CO (ppm)	10	13	0	8
H <sub>2</sub> S (ppm)	4,518	1,478	2,561	2,852
H <sub>2</sub> (ppm)	36	49	106	64

According to Tsai [19], the heating value of biogas (HV<sub>biogas</sub>) calculated for a methane content of 73.93 % in the biogas is 6,777 kcal/m<sup>3</sup>.

**Methane and Biogas Production**

The input data for the model presented in Table III are used to obtain the amount of methane and biogas production according to the equations presented in the model development section.

*TABLE III. Parameters used to obtain the amount of methane and biogas.*

Parameter	<i>Q</i>	<i>S<sub>0</sub></i>	<i>S</i>	<i>T</i>	<i>C<sub>CH4</sub></i>
Unit	m <sup>3</sup> /d	kgCOD <sub>infl</sub> /m <sup>3</sup>	kgCOD <sub>effl</sub> /m <sup>3</sup>	°C	%
Value	200	13.0	0.90	28	73.93

Table IV presents the results obtained for the production of methane and biogas.

*TABLE IV. Results of methane and biogas production*

Parameter	<i>COD<sub>CH4</sub></i>	<i>K(t)</i>	<i>Q<sub>CH4</sub></i>	<i>Q<sub>biogas</sub></i>
Unit	Kg COD <sub>CH4</sub> /d	(kg COD/m <sup>3</sup> )	m <sup>3</sup> /d	m <sup>3</sup> /d
Value	1,978	2.6	761	1,029

**Firewood Consumption**

In the current process eucalyptus firewood is used with an approximate storage of 45 days. Using Table I, with the moisture content of 45.8 %, the firewood heating value (HV<sub>fuel</sub>) can be obtained and is 2,062 kcal/kg.

Table V presents a comparison between the amount of energy consumed (kcal) in the boiler for steam production with the use of firewood and biogas - Situation<sub>1</sub> (*S<sub>1</sub>*), and the energy produced using only firewood – Situation<sub>2</sub> (*S<sub>2</sub>*).

*TABLE V. Comparative between firewood consumption and biogas consumption.*

Comparison	Flowrate <i>S<sub>1</sub></i>	Flowrate <i>S<sub>2</sub></i>	HV <sub>biogas</sub>	HV <sub>fuel</sub>	Energy ( <i>S<sub>1</sub></i> )	Energy ( <i>S<sub>2</sub></i> )
Unit	kg/d	kg/d	kcal/m <sup>3</sup>	kcal/kg	kcal/d	kcal/d
Results	13,000	11,500	6,777	2,062	6,806,000	23,713,100

The difference in the amount of energy used in the boiler between the two situations is 3,093,000 kcal/d. Therefore, the amount of biogas consumed is equivalent to 456 m<sup>3</sup>/d. As the amount of biogas (*Q<sub>biogas</sub>*) produced in the reactors is 1,029 m<sup>3</sup>/d, the stored volume of 573 m<sup>3</sup>/d is the difference between the two quantities.

**Avoided Cost of Firewood**

With the average cost of firewood at \$ 33.74/t and the daily difference expressed in mass of 1.5 t between S<sub>1</sub> and S<sub>2</sub>, the avoided annual cost of firewood is \$ 12,156. If all the biogas produced were used, the avoided cost of firewood would jump from the current 12 % to 26 % and the annual value would be \$ 27,385 as to the avoided cost of firewood.

**Electric Power Consumption**

Fig. 1 presents the one year historic of the industrial process energy consumption (kWh) at peak hours - (from 6:00 p.m. to 9:00 p.m, from Monday to Friday) and out of peak hours (other hours of the day, weekends and holidays) – Brazilian tariffs.

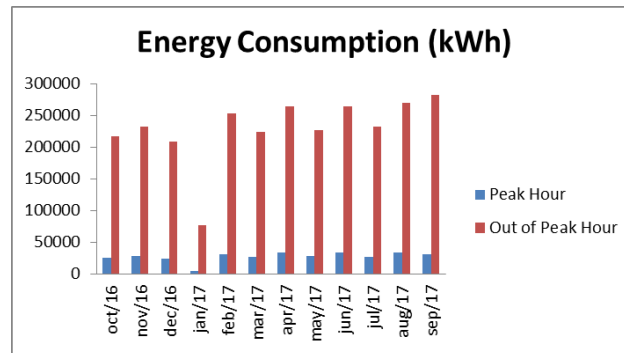


Fig. 1. Energy use profile – Data base electric power company

Total peak hour = 330,818 kWh/y  
 Total out of peak hour = 2,757,492 kWh/y

Table VI presents the average power consumption for one year and the tariffs applied in the last month of evaluation. The company is part of a tax system where it is credited with part (5%) of the tax on the Circulation of Goods and Services, which has a rate of 29 % on the value of electricity.

**TABLE VI. Average energy uses and applied tariff.**

Tariffs	Average (kWh/month)	Cost with tax (\$/kWh)	Cost with tax credit (\$/kWh)
Peak hour	27,568	0.572703	0.544067
Out of peak hour	229,791	0.135842	0.129049

**Avoided Cost of Power**

Some considerations, such as the maximum utilization of the electricity generation system at peak hours (higher energy tariff), presented in Table VII, are important to obtain the avoided cost of electric power.

**TABLE VII. Operation in hours - internal combustion engine.**

Operation Time	Total (h)
Out of peak hour	9,5
Peak hour	3,0

The results refer to the operation of the industrial plant - 24 days a month and 10 months a year. The operation conditions, the results of electric power production and the avoided energy cost are presented in Table VIII, based on the conversion efficiency selected for the application of this model.

**TABLE VIII. Generation and cost of electric power.**

Generation (kWh/d)		Generation (kWh/m)		Generation (kWh/y)	
Out of peak hour	Peak hour	Out of peak hour	Peak hour	Out of peak hour	Peak hour
2,005	633	2,005	48,108	481,080	139,260

Total: 2,638		Total: 62,034		Total: 620,340	
Value (\$)		Value (\$)		Value (\$)	
273	362	7,223	7,975	72,230	79,755
Total: 635		Total: 15198		Total: 151,985	

According to the schedule of revisions, parts and maintenance workmanship of the supplier, one must deduct from the value of the energy generated the corrective and preventive maintenance costs of the internal combustion engine established by the hours of operation.

For the GSCA 330 kVA the base maintenance value is \$ 2.85/h. At this cost, the supplier estimates lifetime of the equipment at 260,000 hours, over an average period of 5 years, including labor services and overhaul of the internal combustion engine. The monthly cost is \$ 855.

Thus, it is possible to generate a total amount of electricity of 62,034 kWh/month, with a cost avoided of \$ 151,985/y.

The energy (P) obtained is 3.19 kW and the investment cost for the implementation of the system for distributed generation of power is \$ 167,337.

Table IX presents the amount of fuel used for the generation of thermal energy without the use of biogas and the annual electricity consumption. From use of biogas as an energy source, it is possible to obtain the reduction percentage of firewood and electric power.

**TABLE IX. Comparison between the avoided consumption of firewood and electricity**

Variable	Firewood (t/y)	Electricity (kWh/y)
Consumption	3,120	3,088,310
Avoided	812	151,985
Consumption		
Reduction (%)	26	5

The industrial plant consumes 330,818 kWh (at higher tariff value) at peak hour and the electricity generated by the biogas would be equivalent to 46 % of this value.

Based on the proposed methodology and its results, in Table 10 are presented the equivalence and the avoided cost of wood for the use of biogas to produce steam and electricity.

**TABLE X: Energy equivalence and avoided costs**

Parameter	Equivalence (Energy/m <sup>3</sup> biogas)	Avoided Cost (\$/y)
Steam Production	3.28 t Firewood	27,385
Electricity Production	2.57 kWh	143,430

The potential available for the production of electricity (*Pot*) obtained is 1,760 kWh/d, about 30 % below the daily amount of electricity presented in Table IX, since the energy equivalence used by the internal combustion engine supplier is 2.57 kWh/m<sup>3</sup>.

The power production with the same amount of biogas available is 47.5 % lower. Considering the average energy conversion [16], and making the correction in the avoided cost of annual electricity, the value would be \$ 75,300. Even so, it is three times the value using biogas for steam production. This difference is significant, even though if the operational and implementation costs of the direct biogas burning system are not considered. The value of the energy tariff at peak hour is 4.2 times higher than the tariff at out peak hour, which has a great influence on the avoided cost of power.

The choice of the 330 kVA internal combustion engine was due to the consumption of the equipment to use all the available biogas, because a larger capacity generator group would operate for a shorter time and produce a greater amount of electricity with the same availability of biogas.

The result of the theoretical biogas production and the quantity consumed in the boiler was consistent with the operational condition and the data provided for the consumption of wood in the boiler, since the burner operates 24 hours a day and the reactor dome remain storing biogas with volume close to that obtained according to the physical dimensions.

At first, it can be observed that the storage of firewood without cover, can influence the consumption in the boiler, due to the changes in its moisture. As the company uses firewood in the boiler always from the oldest stock to the newest, it controls the mass daily and has historical averages. This fact reduces the effect of this possible moisture variation on the annual cost of the avoided firewood.

The heating value is the basis of the energy evaluation and essential for the determination of the return of the investment. The average methane content in the biogas was considered to be 60 %.

## 5. CONCLUSIONS

Residual biomass can be used to produce biogas in industrial processes with organic wastewater. Biogas can be used for the generation of electricity and for thermal energy. It can be used to be directly burnt in in boilers or

jointly with another type of fuel such as firewood, chip, bagasse, diesel oil and natural gas. Its use as fuel also depends on its composition ( $\text{CH}_4$ ,  $\text{CO}_2$ ,  $\text{H}_2\text{S}$ , etc.).

In the present work a mathematical model was developed to estimate the amount of electricity produced using biogas as fuel in association with other fuel. The model was tested in a biotechnology industry, located in Maringá - Brazil. The results allow to evaluate the production of electric power from the use of biogas, in association with firewood. Part of the firewood was replaced and the avoided cost of this second fuel showed to be a very interesting alternative from the economic point of view. It can be concluded that the model is robust and allows to present the amount of electricity that can be generated in industrial processes which can use biogas as fuel.

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